Postscriptum to "On Degrees of Justification"

Gregor Betz*

August 3, 2012

1 Why Equal, Positive Weights?

The degree of justification of a partial position $\mathcal{P}$ is defined as

$$\textsc{Doj}(\mathcal{P}) := \frac{\sigma_\tau(\mathcal{P})}{\sigma_\tau}, \tag{1}$$

where σ_τ is the number of all coherent and complete positions on the dialectical structure τ, and $\sigma_\tau(\mathcal{P})$ refers to the number of all coherent and complete positions on τ that extend $\mathcal{P}$. Let Ω be the set of all complete positions on τ, $\Gamma \subseteq \Omega$ the set of all coherent and complete positions, and $\Gamma(\mathcal{P}) \subseteq \Gamma$ the set of all coherent and complete positions which extend $\mathcal{P}$. With the function $w : \Omega \to \mathbf{R}$, $w(\mathcal{Q}) = c$ for all $\mathcal{Q} \in \Omega$ and some constant $c \in \mathbf{R}^+$, we may express the degree of justification as,

$$\textsc{Doj}(\mathcal{P}) = \frac{\sum_{\mathcal{Q}\in\Gamma(\mathcal{P})} w(\mathcal{Q})}{\sum_{\mathcal{Q}\in\Gamma} w(\mathcal{Q})}. \tag{2}$$

Question is: Why do we set equal weights w for all $\mathcal{Q} \in \Omega$? Couldn't—or, shouldn't—we generalize the definition and introduce $\textsc{Doj}_w$ in line with (2) for arbitrary weights w?

In the following, I'll argue that setting $w(\mathcal{Q})$ constant for all $\mathcal{Q} \in \Omega$ is justified insofar as $\textsc{Doj}$ seeks to explicate our pre-theoretic concept of strength of justification given a certain state of a debate. More precisely, I will claim that we have to set

1. $w(\mathcal{Q}) > 0$ for all $\mathcal{Q} \in \Omega$, since, otherwise, supporting a thesis with a new, independent argument wouldn't necessarily increase its degree of justification;
2. $w(\mathcal{Q}_1) = w(\mathcal{Q}_2)$ for arbitrary $\mathcal{Q}_1, \mathcal{Q}_2 \in \Omega$, as, otherwise, two equally justified theses wouldn't necessarily display the same degree of justification.

Ad 1.: Assume that $w(\mathcal{R}) \leq 0$ some position $\mathcal{R} \in \Omega$. We depict the state of a debate where no arguments have been introduced so far (τ_0); hence $\Gamma_{\tau_0} = \Omega$, and $\mathcal{R}$ is a coherent position. Take three sentences p_1, p_2, c of the debate; without loss of generality (as we may re-label), we assume that p_1, p_2 are true according to $\mathcal{R}$, while c is false. Next, the argument $(p_1, p_2; c)$, which supports c, is introduced into the debate (τ_1). This renders $\mathcal{R}$, and only $\mathcal{R}$, incoherent. As the premisses are not clearly absurd (they are not refuted given background knowledge), it would be

*Karlsruhe Institute of Technology, Germany, email: gregor.betz@kit.edu.

highly counter-intuitive to say that introducing the argument does not increase c's strength of justification. Yet, we have (with the atomic position $\mathcal{P}$ assigning c the value $\mathbf{t}$):

$$\begin{aligned}
\textsc{Doj}_{\tau_0}(\mathcal{P}) &= \frac{\sum_{\mathcal{Q}\in\Gamma_{\tau_0}(\mathcal{P})} w(\mathcal{Q})}{\sum_{\mathcal{Q}\in\Gamma_{\tau_0}} w(\mathcal{Q})} && (3)\\
&= \frac{(\sum_{\mathcal{Q}\in\Gamma_{\tau_1}(\mathcal{P})} w(\mathcal{Q})) + w(\mathcal{R})}{(\sum_{\mathcal{Q}\in\Gamma_{\tau_1}} w(\mathcal{Q})) + w(\mathcal{R})} && (4)\\
&\geq \frac{\sum_{\mathcal{Q}\in\Gamma_{\tau_1}(\mathcal{P})} w(\mathcal{Q})}{\sum_{\mathcal{Q}\in\Gamma_{\tau_1}} w(\mathcal{Q})} && (5)\\
&= \textsc{Doj}_{\tau_1}(\mathcal{P}) && (6)
\end{aligned}$$

Note that (5) holds because $\sum_{\mathcal{Q}\in\Gamma_{\tau_1}(\mathcal{P})} w(\mathcal{Q}) < \sum_{\mathcal{Q}\in\Gamma_{\tau_1}} w(\mathcal{Q})$ and $w(\mathcal{R}) \leq 0$.

Ad 2.: Let $\textsc{Doj}_w$ denote generalized degrees of justification in line with (2). We consider, again, the initial state of the debate where no arguments have been introduced, yet. At this stage, no single claim is any better justified, given the arguments advanced so far, than any other claim. Moreover, no two sentences partially entail each other, given the arguments advanced so far, any more than any two other sentences do. Hence, $\textsc{Doj}_w(s_i) = \textsc{Doj}_w(s_j)$ and $\textsc{Doj}_w(s_i|s_k) = \textsc{Doj}_w(s_j|s_l)$ for arbitrary atomic positions / sentences s_i, s_j, s_k, s_l. As $\textsc{Doj}_w$ satisfies the probability axioms, it follows that not only atomic, but arbitrarily large partial positions of the same size (i.e., defined over the same number of sentences), including complete positions, possess the same $\textsc{Doj}_w$ and, hence, that $w(\mathcal{Q}_1) = w(\mathcal{Q}_2)$ for arbitrary $\mathcal{Q}_1, \mathcal{Q}_2 \in \Omega$.

2 Does the Analysis Rely on the Principle of Indifference?

The principle of indifference states that, in case of complete ignorance, it is rational to assign equal degrees of belief to the corresponding possibilities. It's clear that this principle does not relate to degrees of justification, because degrees of justification don't speak about rational degrees of belief at all.

There is only a formal similarity. Like the principle of indifference, the definition of Doj stipulates that probabilities be assigned uniformly. Yet, the reasons for doing so are completely different. Notably, as argued above, the assignment of equal probabilities/weights to coherent positions is required to capture our inferential practice and to accommodate our argumentative intuitions (concerning strength of justification).